



DIGITAL CIRCUITS

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About the Tutorial

This tutorial is meant to provide the readers to know how to analyze and implement the combinational circuits and sequential circuits. Based on the requirement, we can use either combinational circuit or sequential circuit or combination of both. After completing this tutorial, you will be able to learn the type of digital circuit, which is suitable for specific application.

Audience

This tutorial is meant for all the readers who are aspiring to learn the concepts of digital circuits. Digital circuits contain a set of Logic gates and these can be operated with binary values, 0 and 1.

Prerequisites

A basic idea regarding the initial concepts of Digital Electronics is enough to understand the topics covered in this tutorial.

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Table of Contents

About the Tutorial	1
Audience	1
Prerequisites	1
Copyright & Disclaimer	1
Table of Contents	2
1. DIGITAL CIRCUITS – NUMBER SYSTEMS	7
Decimal Number System	7
Binary Number System	8
Octal Number System	8
Hexadecimal Number System	9
2. DIGITAL CIRCUITS – BASE CONVERSIONS	10
Decimal Number to other Bases Conversion	10
Binary Number to other Bases Conversion	12
Octal Number to other Bases Conversion	14
Hexa-Decimal Number to other Bases Conversion	15
3. DIGITAL CIRCUITS – BINARY NUMBERS REPRESENTATION	17
Representation of Un-Signed Binary Numbers	17
Representation of Signed Binary Numbers	17
4. DIGITAL CIRCUITS – SIGNED BINARY ARITHMETIC	20
Addition of two Signed Binary Numbers	20
Subtraction of two Signed Binary Numbers	21
5. DIGITAL CIRCUITS – CODES	23
Binary Codes for Decimal digits	23
Gray Code	25

6.	DIGITAL CIRCUITS – ERROR DETECTION & CORRECTION CODES.....	27
	Parity Code.....	27
	Hamming Code.....	28
7.	DIGITAL CIRCUITS – BOOLEAN ALGEBRA	31
	Postulates and Basic Laws of Boolean Algebra	31
	Theorems of Boolean Algebra	32
	Simplification of Boolean Functions	33
8.	DIGITAL CIRCUITS – CANONICAL AND STANDARD FORMS	36
	Canonical SoP and PoS forms	36
	Standard SoP and PoS forms	38
9.	DIGITAL CIRCUITS – K-MAP METHOD	41
	K-Maps for 2 to 5 Variables	41
	Minimization of Boolean Functions using K-Maps	43
10.	DIGITAL CIRCUITS – QUINE-MCCLUSKEY TABULAR METHOD.....	47
	Procedure of Quine-McCluskey Tabular Method.....	47
11.	DIGITAL CIRCUITS – LOGIC GATES.....	50
	Basic Gates.....	50
	Universal gates.....	52
	Special Gates.....	54
12.	DIGITAL CIRCUITS – TWO-LEVEL LOGIC REALIZATION	56
	Degenerative Form.....	56
	Non-degenerative Form	58

13. DIGITAL CIRCUITS – COMBINATIONAL CIRCUITS.....	61
Code Converters.....	61
Parity Bit Generator	64
Parity Checker	65
14. DIGITAL CIRCUITS – ARITHMETIC CIRCUITS	68
Binary Adder	68
Binary Subtractor	71
Binary Adder / Subtractor	72
15. DIGITAL CIRCUITS – DECODERS	74
2 to 4 Decoder.....	74
Implementation of Higher-order Decoders	75
16. DIGITAL CIRCUITS – ENCODERS	78
4 to 2 Encoder	78
Octal to Binary Encoder.....	79
Priority Encoder	81
17. DIGITAL CIRCUITS – MULTIPLEXERS.....	83
4x1 Multiplexer.....	83
Implementation of Higher-order Multiplexers	84
18. DIGITAL CIRCUITS – DE-MULTIPLEXERS	88
1x4 De-Multiplexer	88
Implementation of Higher-order De-Multiplexers.....	89
19. DIGITAL CIRCUITS – PROGRAMMABLE LOGIC DEVICES.....	92
Programmable Read Only Memory (PROM).....	92
Programmable Array Logic (PAL)	93
Programmable Logic Array (PLA)	95

20. DIGITAL CIRCUITS – THRESHOLD LOGIC.....	97
Basics of Threshold gate	97
Synthesis of Threshold Functions	99
21. DIGITAL CIRCUITS – SEQUENTIAL CIRCUITS	102
Types of Sequential Circuits	103
Clock Signal and Triggering	103
22. DIGITAL CIRCUITS – LATCHES	106
SR Latch	106
D Latch	107
23. DIGITAL CIRCUITS – FLIP-FLOPS	109
SR Flip-Flop	109
D Flip-Flop	111
JK Flip-Flop	112
T Flip-Flop	113
24. CONVERSION OF FLIP-FLOPS	115
SR Flip-Flop to other Flip-Flop Conversions	115
D Flip-Flop to other Flip-Flop Conversions	117
JK Flip-Flop to other Flip-Flop Conversions	118
T Flip-Flop to other Flip-Flop Conversions	120
25. DIGITAL CIRCUITS – SHIFT REGISTERS.....	121
Serial In - Serial Out (SISO) Shift Register	121
Serial In - Parallel Out (SIPO) Shift Register	122
Parallel In - Serial Out (PISO) Shift Register	123
Parallel In - Parallel Out (PIPO) Shift Register	124

26. DIGITAL CIRCUITS – APPLICATION OF SHIFT REGISTERS.....	126
Ring Counter	126
Johnson Ring Counter	127
27. DIGITAL CIRCUITS – COUNTERS.....	129
Asynchronous Counters	129
Synchronous Counters	132
28. DIGITAL CIRCUITS – FINITE STATE MACHINES.....	134
Mealy State Machine	134
Moore State Machine	135
29. DIGITAL CIRCUITS – ALGORITHMIC STATE MACHINE CHARTS.....	137
Basic Components of ASM charts	137

1. Digital Circuits – Number Systems

If base or radix of a number system is 'r', then the numbers present in that number system are ranging from zero to r-1. The total numbers present in that number system is 'r'. So, we will get various number systems, by choosing the values of radix as greater than or equal to two.

In this chapter, let us discuss about the **popular number systems** and how to represent a number in the respective number system. The following number systems are the most commonly used.

- Decimal Number system
- Binary Number system
- Octal Number system
- Hexadecimal Number system

Decimal Number System

The **base** or radix of Decimal number system is **10**. So, the numbers ranging from 0 to 9 are used in this number system. The part of the number that lies to the left of the **decimal point** is known as integer part. Similarly, the part of the number that lies to the right of the decimal point is known as fractional part.

In this number system, the successive positions to the left of the decimal point having weights of 10^0 , 10^1 , 10^2 , 10^3 and so on. Similarly, the successive positions to the right of the decimal point having weights of 10^{-1} , 10^{-2} , 10^{-3} and so on. That means, each position has specific weight, which is **power of base 10**.

Example

Consider the **decimal number 1358.246**. Integer part of this number is 1358 and fractional part of this number is 0.246. The digits 8, 5, 3 and 1 have weights of 10^0 , 10^1 , 10^2 and 10^3 respectively. Similarly, the digits 2, 4 and 6 have weights of 10^{-1} , 10^{-2} and 10^{-3} respectively.

Mathematically, we can write it as

$$1358.246 = (1 \times 10^3) + (3 \times 10^2) + (5 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (4 \times 10^{-2}) + (6 \times 10^{-3})$$

After simplifying the right hand side terms, we will get the decimal number, which is on left hand side.

Binary Number System

All digital circuits and systems use this binary number system. The **base** or radix of this number system is **2**. So, the numbers 0 and 1 are used in this number system.

The part of the number, which lies to the left of the **binary point** is known as integer part. Similarly, the part of the number, which lies to the right of the binary point is known as fractional part.

In this number system, the successive positions to the left of the binary point having weights of 2^0 , 2^1 , 2^2 , 2^3 and so on. Similarly, the successive positions to the right of the binary point having weights of 2^{-1} , 2^{-2} , 2^{-3} and so on. That means, each position has specific weight, which is **power of base 2**.

Example

Consider the **binary number 1101.011**. Integer part of this number is 1101 and fractional part of this number is 0.011. The digits 1, 0, 1 and 1 of integer part have weights of 2^0 , 2^1 , 2^2 and 2^3 respectively. Similarly, the digits 0, 1 and 1 of fractional part have weights of 2^{-1} , 2^{-2} and 2^{-3} respectively.

Mathematically, we can write it as

$$1101.011 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$$

After simplifying the right hand side terms, we will get a decimal number, which is an equivalent of binary number on left hand side.

Octal Number System

The **base** or radix of octal number system is **8**. So, the numbers ranging from 0 to 7 are used in this number system. The part of the number that lies to the left of the **octal point** is known as integer part. Similarly, the part of the number that lies to the right of the octal point is known as fractional part.

In this number system, the successive positions to the left of the octal point having weights of 8^0 , 8^1 , 8^2 , 8^3 and so on. Similarly, the successive positions to the right of the octal point having weights of 8^{-1} , 8^{-2} , 8^{-3} and so on. That means, each position has specific weight, which is **power of base 8**.

Example

Consider the **octal number 1457.236**. Integer part of this number is 1457 and fractional part of this number is 0.236. The digits 7, 5, 4 and 1 have weights of 8^0 , 8^1 , 8^2 and 8^3 respectively. Similarly, the digits 2, 3 and 6 have weights of 8^{-1} , 8^{-2} and 8^{-3} respectively.

Mathematically, we can write it as

$$1457.236 = (1 \times 8^3) + (4 \times 8^2) + (5 \times 8^1) + (7 \times 8^0) + (2 \times 8^{-1}) + (3 \times 8^{-2}) + (6 \times 8^{-3})$$

After simplifying the right hand side terms, we will get a decimal number, which is an equivalent of octal number on left hand side.

Hexadecimal Number System

The **base** or radix of Hexa-decimal number system is **16**. So, the numbers ranging from 0 to 9 and the letters from A to F are used in this number system. The decimal equivalent of Hexa-decimal digits from A to F are 10 to 15.

The part of the number, which lies to the left of the **hexadecimal point** is known as integer part. Similarly, the part of the number, which lies to the right of the Hexa-decimal point is known as fractional part.

In this number system, the successive positions to the left of the Hexa-decimal point having weights of 16^0 , 16^1 , 16^2 , 16^3 and so on. Similarly, the successive positions to the right of the Hexa-decimal point having weights of 16^{-1} , 16^{-2} , 16^{-3} and so on. That means, each position has specific weight, which is **power of base 16**.

Example

Consider the **Hexa-decimal number 1A05.2C4**. Integer part of this number is 1A05 and fractional part of this number is 0.2C4. The digits 5, 0, A and 1 have weights of 16^0 , 16^1 , 16^2 and 16^3 respectively. Similarly, the digits 2, C and 4 have weights of 16^{-1} , 16^{-2} and 16^{-3} respectively.

Mathematically, we can write it as

$$1A05.2C4 = (1 \times 16^3) + (10 \times 16^2) + (0 \times 16^1) + (5 \times 16^0) + (2 \times 16^{-1}) + (12 \times 16^{-2}) + (4 \times 16^{-3})$$

After simplifying the right hand side terms, we will get a decimal number, which is an equivalent of Hexa-decimal number on left hand side.

2. Digital Circuits – Base Conversions

In previous chapter, we have seen the four prominent number systems. In this chapter, let us convert the numbers from one number system to the other in order to find the equivalent value.

Decimal Number to other Bases Conversion

If the decimal number contains both integer part and fractional part, then convert both the parts of decimal number into other base individually. Follow these steps for converting the decimal number into its equivalent number of any base 'r'.

- Do **division** of integer part of decimal number and **successive quotients** with base 'r' and note down the remainders till the quotient is zero. Consider the remainders in reverse order to get the integer part of equivalent number of base 'r'. That means, first and last remainders denote the least significant digit and most significant digit respectively.
- Do **multiplication** of fractional part of decimal number and **successive fractions** with base 'r' and note down the carry till the result is zero or the desired number of equivalent digits is obtained. Consider the normal sequence of carry in order to get the fractional part of equivalent number of base 'r'.

Decimal to Binary Conversion

The following two types of operations take place, while converting decimal number into its equivalent binary number.

- Division of integer part and successive quotients with base 2.
- Multiplication of fractional part and successive fractions with base 2.

Example

Consider the **decimal number 58.25**. Here, the integer part is 58 and fractional part is 0.25.

Step1: Division of 58 and successive quotients with base 2.

Operation	Quotient	Remainder
58/2	29	0 (LSB)
29/2	14	1
14/2	7	0
7/2	3	1
3/2	1	1
1/2	0	1 (MSB)

$$\Rightarrow (58)_{10} = (111010)_2$$

Therefore, the **integer part** of equivalent binary number is **111010**.

Step 2: Multiplication of 0.25 and successive fractions with base 2.

Operation	Result	Carry
0.25×2	0.5	0
0.5×2	1.0	1
-	0.0	-

$$\Rightarrow (0.25)_{10} = (01)_{2}$$

Therefore, the **fractional part** of equivalent binary number is **.01**.

$$\Rightarrow (58.25)_{10} = (111010.01)_{2}$$

Therefore, the **binary equivalent** of decimal number 58.25 is 111010.01.

Decimal to Octal Conversion

The following two types of operations take place, while converting decimal number into its equivalent octal number.

- Division of integer part and successive quotients with base 8.
- Multiplication of fractional part and successive fractions with base 8.

Example

Consider the **decimal number 58.25**. Here, the integer part is 58 and fractional part is 0.25.

Step 1: Division of 58 and successive quotients with base 8.

Operation	Quotient	Remainder
$58/8$	7	2
$7/8$	0	7

$$\Rightarrow (58)_{10} = (72)_{8}$$

Therefore, the **integer part** of equivalent octal number is **72**.

Step 2: Multiplication of 0.25 and successive fractions with base 8.

Operation	Result	Carry
0.25×8	2.00	2
-	0.00	-

$$\Rightarrow (0.25)_{10} = (2)_{8}$$

Therefore, the **fractional part** of equivalent octal number is **.2**.

$$\Rightarrow (58.25)_{10} = (72.2)_{8}$$

Therefore, the **octal equivalent** of decimal number 58.25 is 72.2.

Decimal to Hexa-Decimal Conversion

The following two types of operations take place, while converting decimal number into its equivalent hexa-decimal number.

- Division of integer part and successive quotients with base 16.
- Multiplication of fractional part and successive fractions with base 16.

Example

Consider the **decimal number 58.25**. Here, the integer part is 58 and decimal part is 0.25.

Step 1: Division of 58 and successive quotients with base 16.

Operation	Quotient	Remainder
58/16	3	10=A
3/16	0	3

$$\Rightarrow (58)_{10} = (3A)_{16}$$

Therefore, the **integer part** of equivalent Hexa-decimal number is 3A.

Step 2: Multiplication of 0.25 and successive fractions with base 16.

Operation	Result	Carry
0.25 x 16	4.00	4
-	0.00	-

$$\Rightarrow (\cancel{.}25)_{10} = (\cancel{.}4)_{16}$$

Therefore, the **fractional part** of equivalent Hexa-decimal number is .4.

$$\Rightarrow (58\cancel{.}25)_{10} = (3A\cancel{.}4)_{16}$$

Therefore, the **Hexa-decimal equivalent** of decimal number 58.25 is 3A.4.

Binary Number to other Bases Conversion

The process of converting a number from binary to decimal is different to the process of converting a binary number to other bases. Now, let us discuss about the conversion of a binary number to decimal, octal and Hexa-decimal number systems one by one.

Binary to Decimal Conversion

For converting a binary number into its equivalent decimal number, first multiply the bits of binary number with the respective positional weights and then add all those products.

Example

Consider the **binary number 1101.11**.

Mathematically, we can write it as

$$\begin{aligned}(1101.11)_2 &= (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2}) \\ \Rightarrow (1101.11)_2 &= 8 + 4 + 0 + 1 + 0.5 + 0.25 = 13.75 \\ \Rightarrow (1101.11)_2 &= (13.75)_{10}\end{aligned}$$

Therefore, the **decimal equivalent** of binary number 1101.11 is 13.75.

Binary to Octal Conversion

We know that the bases of binary and octal number systems are 2 and 8 respectively. Three bits of binary number is equivalent to one octal digit, since $2^3 = 8$.

Follow these two steps for converting a binary number into its equivalent octal number.

- Start from the binary point and make the groups of 3 bits on both sides of binary point. If one or two bits are less while making the group of 3 bits, then include required number of zeros on extreme sides.
- Write the octal digits corresponding to each group of 3 bits.

Example

Consider the **binary number 101110.01101**.

Step 1: Make the groups of 3 bits on both sides of binary point.

$$101\ 110.011\ 01$$

Here, on right side of binary point, the last group is having only 2 bits. So, include one zero on extreme side in order to make it as group of 3 bits.

$$\Rightarrow 101\ 110.011\ 010$$

Step 2: Write the octal digits corresponding to each group of 3 bits.

$$\Rightarrow (101\ 110.011\ 010)_2 = (56.32)_8$$

Therefore, the **octal equivalent** of binary number 101110.01101 is 56.32.

Binary to Hexa-Decimal Conversion

We know that the bases of binary and Hexa-decimal number systems are 2 and 16 respectively. Four bits of binary number is equivalent to one Hexa-decimal digit, since $2^4 = 16$.

Follow these two steps for converting a binary number into its equivalent Hexa-decimal number.

- Start from the binary point and make the groups of 4 bits on both sides of binary point. If some bits are less while making the group of 4 bits, then include required number of zeros on extreme sides.

- Write the Hexa-decimal digits corresponding to each group of 4 bits.

Example

Consider the **binary number 101110.01101**.

Step 1: Make the groups of 4 bits on both sides of binary point.

$$10\ 1110.0110\ 1$$

Here, the first group is having only 2 bits. So, include two zeros on extreme side in order to make it as group of 4 bits. Similarly, include three zeros on extreme side in order to make the last group also as group of 4 bits.

$$\Rightarrow 0010\ 1110.0110\ 1000$$

Step 2: Write the Hexa-decimal digits corresponding to each group of 4 bits.

$$\Rightarrow (0010\ 1110.0110\ 1000)_2 = (2E.68)_{16}$$

Therefore, the **Hexa-decimal equivalent** of binary number 101110.01101 is 2E.68.

Octal Number to other Bases Conversion

The process of converting a number from octal to decimal is different to the process of converting an octal number to other bases. Now, let us discuss about the conversion of an octal number to decimal, binary and Hexa-decimal number systems one by one.

Octal to Decimal Conversion

For converting an octal number into its equivalent decimal number, first multiply the digits of octal number with the respective positional weights and then add all those products.

Example

Consider the **octal number 145.23**.

Mathematically, we can write it as

$$(145.23)_8 = (1 \times 8^2) + (4 \times 8^1) + (5 \times 8^0) + (2 \times 8^{-1}) + (3 \times 8^{-2})$$

$$\Rightarrow (145.23)_8 = 64 + 32 + 5 + 0.25 + 0.05 = 101.3$$

$$\Rightarrow (145.23)_8 = (101.3)_{10}$$

Therefore, the **decimal equivalent** of octal number 145.23 is 101.3.

Octal to Binary Conversion

The process of converting an octal number to an equivalent binary number is just opposite to that of binary to octal conversion. By representing each octal digit with 3 bits, we will get the equivalent binary number.

Example

Consider the **octal number 145.23**.

Represent each octal digit with 3 bits.

$$(145.23)_8 = (001\ 100\ 101.010\ 011)_2$$

The value doesn't change by removing the zeros, which are on the extreme side.

$$\Rightarrow (145.23)_8 = (1100101.010011)_2$$

Therefore, the **binary equivalent** of octal number 145.23 is 1100101.010011.

Octal to Hexa-Decimal Conversion

Follow these two steps for converting an octal number into its equivalent Hexa-decimal number.

- Convert octal number into its equivalent binary number.
- Convert the above binary number into its equivalent Hexa-decimal number.

Example

Consider the **octal number 145.23**.

In previous example, we got the binary equivalent of octal number 145.23 as 1100101.010011.

By following the procedure of binary to Hexa-decimal conversion, we will get

$$(1100101.010011)_2 = (65.4C)_{16}$$

$$\Rightarrow (145.23)_8 = (65.4C)_{16}$$

Therefore, the **Hexa-decimal equivalent** of octal number 145.23 is 65.4C.

Hexa-Decimal Number to other Bases Conversion

The process of converting a number from Hexa-decimal to decimal is different to the process of converting Hexa-decimal number into other bases. Now, let us discuss about the conversion of Hexa-decimal number to decimal, binary and octal number systems one by one.

Hexa-Decimal to Decimal Conversion

For converting Hexa-decimal number into its equivalent decimal number, first multiply the digits of Hexa-decimal number with the respective positional weights and then add all those products.

Example

Consider the **Hexa-decimal number 1A5.2**.

Mathematically, we can write it as

$$\begin{aligned}(1A5.2)_{16} &= (1 \times 16^2) + (10 \times 16^1) + (5 \times 16^0) + (2 \times 16^{-1}) \\ \Rightarrow (1A5.2)_{16} &= 256 + 160 + 5 + 0.125 = 421.125 \\ \Rightarrow (1A5.2)_{16} &= (421.125)_{10}\end{aligned}$$

Therefore, the **decimal equivalent** of Hexa-decimal number 1A5.2 is 421.125.

Hexa-Decimal to Binary Conversion

The process of converting Hexa-decimal number into its equivalent binary number is just opposite to that of binary to Hexa-decimal conversion. By representing each Hexa-decimal digit with 4 bits, we will get the equivalent binary number.

Example

Consider the **Hexa-decimal number 65.4C**.

Represent each Hexa-decimal digit with 4 bits.

$$(65.4C)_{16} = (0110\ 0101.0100\ 1100)_2$$

The value doesn't change by removing the zeros, which are at two extreme sides.

$$\Rightarrow (65.4C)_{16} = (1100101.010011)_2$$

Therefore, the **binary equivalent** of Hexa-decimal number 65.4C is 1100101.010011.

Hexa-Decimal to Octal Conversion

Follow these two steps for converting Hexa-decimal number into its equivalent octal number.

- Convert Hexa-decimal number into its equivalent binary number.
- Convert the above binary number into its equivalent octal number.

Example

Consider the **Hexa-decimal number 65.4C**.

In previous example, we got the binary equivalent of Hexa-decimal number 65.4C as 1100101.010011.

By following the procedure of binary to octal conversion, we will get

$$(1100101.010011)_2 = (145.23)_8$$

$$\Rightarrow (65.4C)_{16} = (145.23)_8$$

Therefore, the **octal equivalent** of Hexa-decimal number 65.4C is 145.23.

3. Digital Circuits – Binary Numbers Representation

We can make the binary numbers into the following two groups: **Unsigned numbers** and **Signed numbers**.

Unsigned Numbers

Unsigned numbers contain only magnitude of the number. They don't have any sign. That means all unsigned binary numbers are positive. As in decimal number system, the placing of positive sign in front of the number is optional for representing positive numbers. Therefore, all positive numbers including zero can be treated as unsigned numbers if positive sign is not assigned in front of the number.

Signed Numbers

Signed numbers contain both sign and magnitude of the number. Generally, the sign is placed in front of number. So, we have to consider the positive sign for positive numbers and negative sign for negative numbers. Therefore, all numbers can be treated as signed numbers if the corresponding sign is assigned in front of the number.

If sign bit is zero, which indicates the binary number is positive. Similarly, if sign bit is one, which indicates the binary number is negative.

Representation of Un-Signed Binary Numbers

The bits present in the un-signed binary number holds the **magnitude** of a number. That means, if the un-signed binary number contains 'N' bits, then all N bits represent the magnitude of the number, since it doesn't have any sign bit.

Example

Consider the **decimal number 108**. The binary equivalent of this number is **1101100**. This is the representation of unsigned binary number.

$$(108)_{10} = (1101100)_2$$

It is having 7 bits. These 7 bits represent the magnitude of the number 108.

Representation of Signed Binary Numbers

The Most Significant Bit (MSB) of signed binary numbers is used to indicate the sign of the numbers. Hence, it is also called as **sign bit**. The positive sign is represented by placing '0' in the sign bit. Similarly, the negative sign is represented by placing '1' in the sign bit.

If the signed binary number contains 'N' bits, then (N-1) bits only represent the magnitude of the number since one bit (MSB) is reserved for representing sign of the number.

There are three **types of representations** for signed binary numbers.

- Sign-Magnitude form
- 1's complement form
- 2's complement form

Representation of a positive number in all these 3 forms is same. But, only the representation of negative number will differ in each form.

Example

Consider the **positive decimal number +108**. The binary equivalent of magnitude of this number is 1101100. These 7 bits represent the magnitude of the number 108. Since it is positive number, consider the sign bit as zero, which is placed on left most side of magnitude.

$$(+108)_{10} = (01101100)_2$$

Therefore, the **signed binary representation** of positive decimal number +108 is **01101100**. So, the same representation is valid in sign-magnitude form, 1's complement form and 2's complement form for positive decimal number +108.

Sign-Magnitude form

In sign-magnitude form, the MSB is used for representing **sign** of the number and the remaining bits represent the **magnitude** of the number. So, just include sign bit at the left most side of unsigned binary number. This representation is similar to the signed decimal numbers representation.

Example

Consider the **negative decimal number -108**. The magnitude of this number is 108. We know the unsigned binary representation of 108 is 1101100. It is having 7 bits. All these bits represent the magnitude.

Since the given number is negative, consider the sign bit as one, which is placed on left most side of magnitude.

$$(-108)_{10} = (11101100)_2$$

Therefore, the sign-magnitude representation of -108 is **11101100**.

1's complement form

The 1's complement of a number is obtained by **complementing all the bits** of signed binary number. So, 1's complement of positive number gives a negative number. Similarly, 1's complement of negative number gives a positive number.

That means, if you perform two times 1's complement of a binary number including sign bit, then you will get the original signed binary number.

Example

Consider the **negative decimal number -108**. The magnitude of this number is 108. We know the signed binary representation of 108 is 01101100.

It is having 8 bits. The MSB of this number is zero, which indicates positive number. Complement of zero is one and vice-versa. So, replace zeros by ones and ones by zeros in order to get the negative number.

$$(-108)_{10} = (10010011)_2$$

Therefore, the **1's complement of (108)₁₀** is **(10010011)₂**.

2's complement form

The 2's complement of a binary number is obtained by **adding one to the 1's complement** of signed binary number. So, 2's complement of positive number gives a negative number. Similarly, 2's complement of negative number gives a positive number.

That means, if you perform two times 2's complement of a binary number including sign bit, then you will get the original signed binary number.

Example

Consider the **negative decimal number -108**.

We know the 1's complement of **(108)₁₀** is **(10010011)₂**.

$$\begin{aligned} 2's \text{ complement of } (108)_{10} &= 1's \text{ complement of } (108)_{10} + 1. \\ &= 10010011 + 1 \\ &= 10010100 \end{aligned}$$

Therefore, the **2's complement of (108)₁₀** is **(10010100)₂**.

4. Digital Circuits – Signed Binary Arithmetic

In this chapter, let us discuss about the basic arithmetic operations, which can be performed on any two signed binary numbers using 2's complement method. The **basic arithmetic operations** are addition and subtraction.

Addition of two Signed Binary Numbers

Consider the two signed binary numbers A & B, which are represented in 2's complement form. We can perform the **addition** of these two numbers, which is similar to the addition of two unsigned binary numbers. But, if the resultant sum contains carry out from sign bit, then discard (ignore) it in order to get the correct value.

If resultant sum is positive, you can find the magnitude of it directly. But, if the resultant sum is negative, then take 2's complement of it in order to get the magnitude.

Example 1

Let us perform the **addition of** two decimal numbers **+7 and +4** using 2's complement method.

The **2's complement** representations of +7 and +4 with 5 bits each are shown below.

$$(+7)_{10} = (00111)_2$$

$$(+4)_{10} = (00100)_2$$

The addition of these two numbers is

$$\begin{aligned} (+7)_{10} + (+4)_{10} &= (00111)_2 + (00100)_2 \\ \Rightarrow (+7)_{10} + (+4)_{10} &= (01011)_2. \end{aligned}$$

The resultant sum contains 5 bits. So, there is no carry out from sign bit. The sign bit '0' indicates that the resultant sum is **positive**. So, the magnitude of sum is 11 in decimal number system. Therefore, addition of two positive numbers will give another positive number.

Example 2

Let us perform the **addition of** two decimal numbers **-7 and -4** using 2's complement method.

The **2's complement** representation of -7 and -4 with 5 bits each are shown below.

$$(-7)_{10} = (11001)_2$$

$$(-4)_{10} = (11100)_2$$

The addition of these two numbers is

$$\begin{aligned} (-7)_{10} + (-4)_{10} &= (11001)_2 + (11100)_2 \\ \Rightarrow (-7)_{10} + (-4)_{10} &= (110101)_2. \end{aligned}$$

The resultant sum contains 6 bits. In this case, carry is obtained from sign bit. So, we can remove it.

Resultant sum after removing carry is $(-7)_{10} + (-4)_{10} = (10101)_2$.

The sign bit '1' indicates that the resultant sum is **negative**. So, by taking 2's complement of it we will get the magnitude of resultant sum as 11 in decimal number system. Therefore, addition of two negative numbers will give another negative number.

Subtraction of two Signed Binary Numbers

Consider the two signed binary numbers A & B, which are represented in 2's complement form. We know that 2's complement of positive number gives a negative number. So, whenever we have to subtract a number B from number A, then take 2's complement of B and add it to A. So, **mathematically** we can write it as

$$A - B = A + (2's \text{ complement of } B)$$

Similarly, if we have to subtract the number A from number B, then take 2's complement of A and add it to B. So, **mathematically** we can write it as

$$B - A = B + (2's \text{ complement of } A)$$

So, the subtraction of two signed binary numbers is similar to the addition of two signed binary numbers. But, we have to take 2's complement of the number, which is supposed to be subtracted. This is the **advantage** of 2's complement technique. Follow, the same rules of addition of two signed binary numbers.

Example 3

Let us perform the **subtraction** of two decimal numbers **+7 and +4** using 2's complement method.

The subtraction of these two numbers is

$$(+7)_{10} - (+4)_{10} = (+7)_{10} + (-4)_{10}$$

The **2's complement** representation of +7 and -4 with 5 bits each are shown below.

$$(+7)_{10} = (00111)_2$$

$$(-4)_{10} = (11100)_2$$

$$\Rightarrow (+7)_{10} + (-4)_{10} = (00111)_2 + (11100)_2 = (100011)_2$$

Here, the carry obtained from sign bit. So, we can remove it. The resultant sum after removing carry is

$$(+7)_{10} + (-4)_{10} = (00011)_2$$

The sign bit '0' indicates that the resultant sum is **positive**. So, the magnitude of it is 3 in decimal number system. Therefore, subtraction of two decimal numbers +7 and +4 is +3.

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