GRAPH THEORY

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About the Tutorial

This tutorial offers a brief introduction to the fundamentals of graph theory. Written in a reader-friendly style, it covers the types of graphs, their properties, trees, graph traversability, and the concepts of coverings, coloring, and matching.

Audience

This tutorial has been designed for students who want to learn the basics of Graph Theory. Graph Theory has a wide range of applications in engineering and hence, this tutorial will be quite useful for readers who are into Language Processing or Computer Networks, physical sciences and numerous other fields.

Prerequisites

Before you start with this tutorial, you need to know elementary number theory and basic set operations in Mathematics. It is mandatory to have a basic knowledge of Computer Science as well.

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Table of Contents

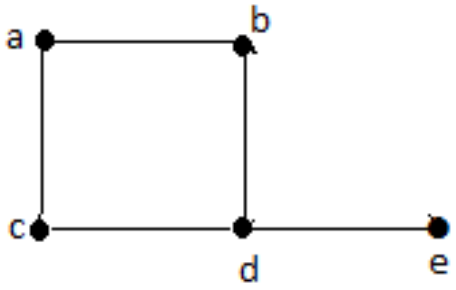
1. GRAPH THEORY – INTRODUCTION

In the domain of mathematics and computer science, *graph theory is the study of graphs that concerns with the relationship among edges and vertices.* It is a popular subject having its applications in computer science, information technology, biosciences, mathematics, and linguistics to name a few. Without further ado, let us start with defining a graph.

What is a Graph?

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as **vertices**, and the links that connect the vertices are called **edges**.

Formally, a graph is a pair of sets **(V, E)**, where **V** is the set of vertices and **E** is the set of edges, connecting the pairs of vertices. Take a look at the following graph:



In the above graph,

V = {a, b, c, d, e}

E = {ab, ac, bd, cd, de}

Applications of Graph Theory

Graph theory has its applications in diverse fields of engineering:

- **Electrical Engineering** – The concepts of graph theory is used extensively in designing circuit connections. The types or organization of connections are named as topologies. Some examples for topologies are star, bridge, series, and parallel topologies.

- **Computer Science** – Graph theory is used for the study of algorithms. For example,

  - Kruskal's Algorithm
  - Prim's Algorithm
  - Dijkstra's Algorithm

- **Computer Network** – The relationships among interconnected computers in the network follows the principles of graph theory.

- **Science** – The molecular structure and chemical structure of a substance, the DNA structure of an organism, etc., are represented by graphs.

- **Linguistics** – The parsing tree of a language and grammar of a language uses graphs.

- **General** – Routes between the cities can be represented using graphs. Depicting hierarchical ordered information such as family tree can be used as a special type of graph called tree.

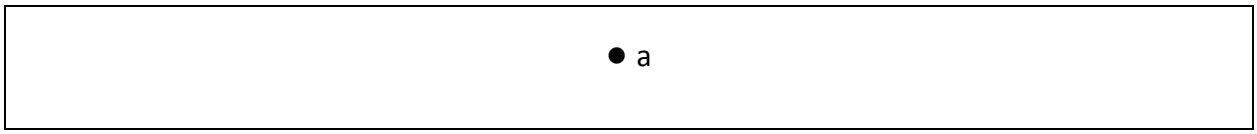
2. GRAPH THEORY – FUNDAMENTALS

A graph is a diagram of points and lines connected to the points. It has at least one line joining a set of two vertices with no vertex connecting itself. The concept of graphs in graph theory stands up on some basic terms such as point, line, vertex, edge, degree of vertices, properties of graphs, etc. Here, in this chapter, we will cover these fundamentals of graph theory.

Point

A point is a particular position in a one-dimensional, two-dimensional, or three-dimensional space. For better understanding, a point can be denoted by an alphabet. It can be represented with a dot.

Example

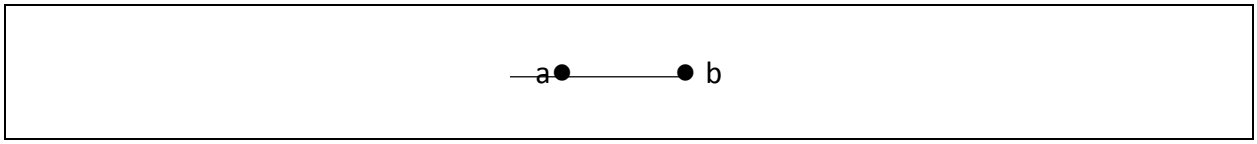


Here, the dot is a point named 'a'.

Line

A **Line** is a connection between two points. It can be represented with a solid line.

Example

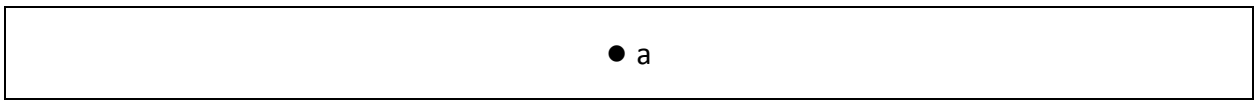


Here, 'a' and 'b' are the points. The link between these two points is called a line.

Vertex

A vertex is a point where multiple lines meet. It is also called a **node**. Similar to points, a vertex is also denoted by an alphabet.

Example

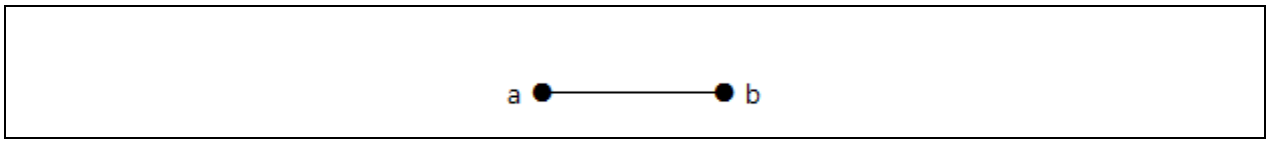


Here, the vertex is named with an alphabet 'a'.

Edge

An edge is the mathematical term for a line that connects two vertices. Many edges can be formed from a single vertex. Without a vertex, an edge cannot be formed. There must be a starting vertex and an ending vertex for an edge.

Example

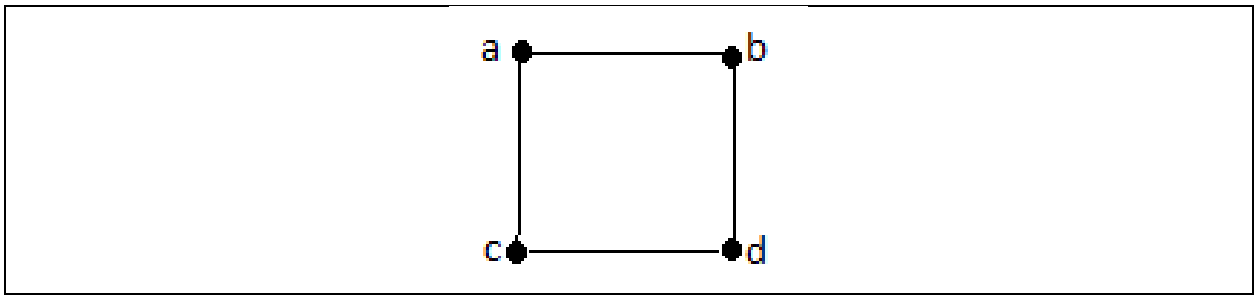


Here, 'a' and 'b' are the two vertices and the link between them is called an edge.

Graph

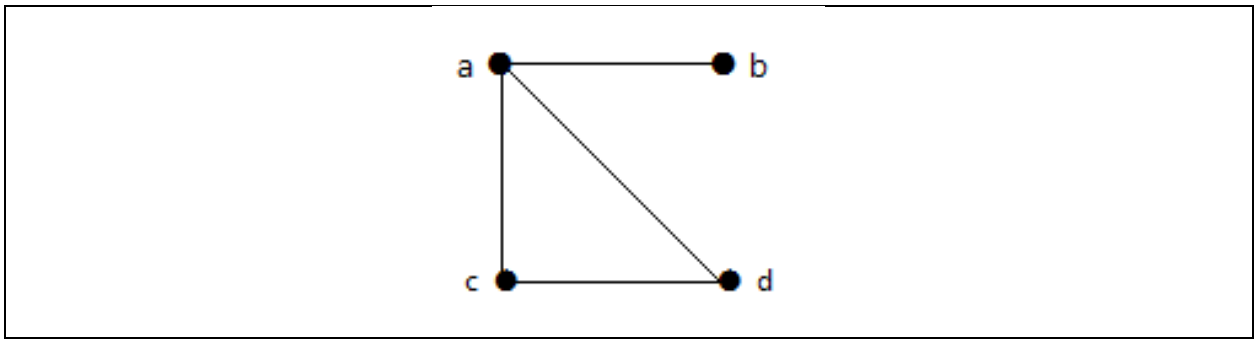
A graph 'G' is defined as G = (V, E) Where V is a set of all vertices and E is a set of all edges in the graph.

Example 1



In the above example, ab, ac, cd, and bd are the edges of the graph. Similarly, a, b, c, and d are the vertices of the graph.

Example 2

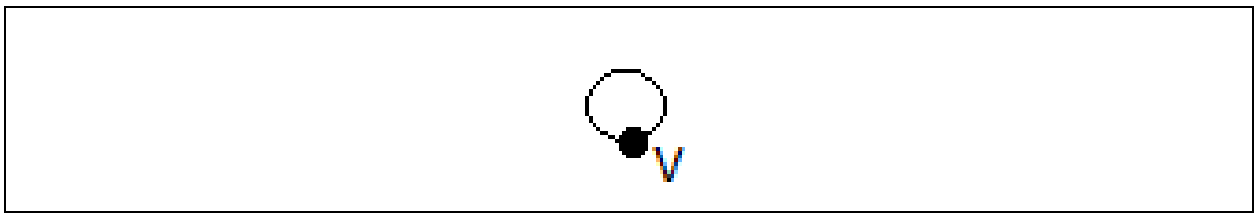


In this graph, there are four vertices a, b, c, and d, and four edges ab, ac, ad, and cd.

Loop

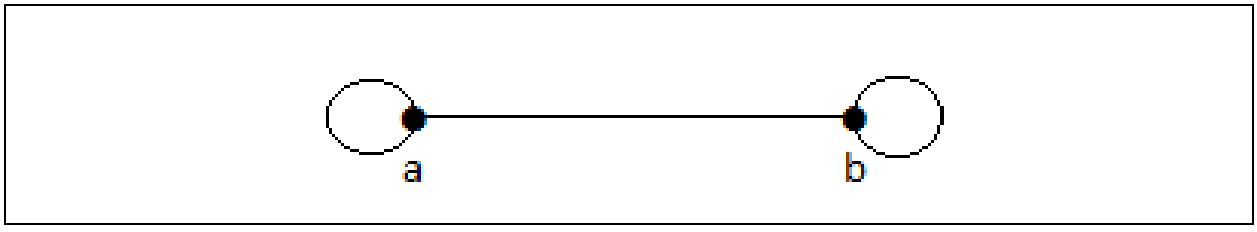
In a graph, if an edge is drawn from vertex to itself, it is called a loop.

Example 1



In the above graph, V is a vertex for which it has an edge (V, V) forming a loop.

Example 2



In this graph, there are two loops which are formed at vertex a, and vertex b.

Degree of Vertex

It is the number of vertices incident with the vertex V.
Notation: deg(V).

In a simple graph with *n* number of vertices, the degree of any vertices is:

$$\deg(v) \leq n - 1 \ \forall \ \ v \in G$$

A vertex can form an edge with all other vertices except by itself. So the degree of a vertex will be up to the **number of vertices in the graph minus 1**. This 1 is for the self-vertex as it cannot form a loop by itself. If there is a loop at any of the vertices, then it is not a Simple Graph.

Degree of vertex can be considered under two cases of graphs:

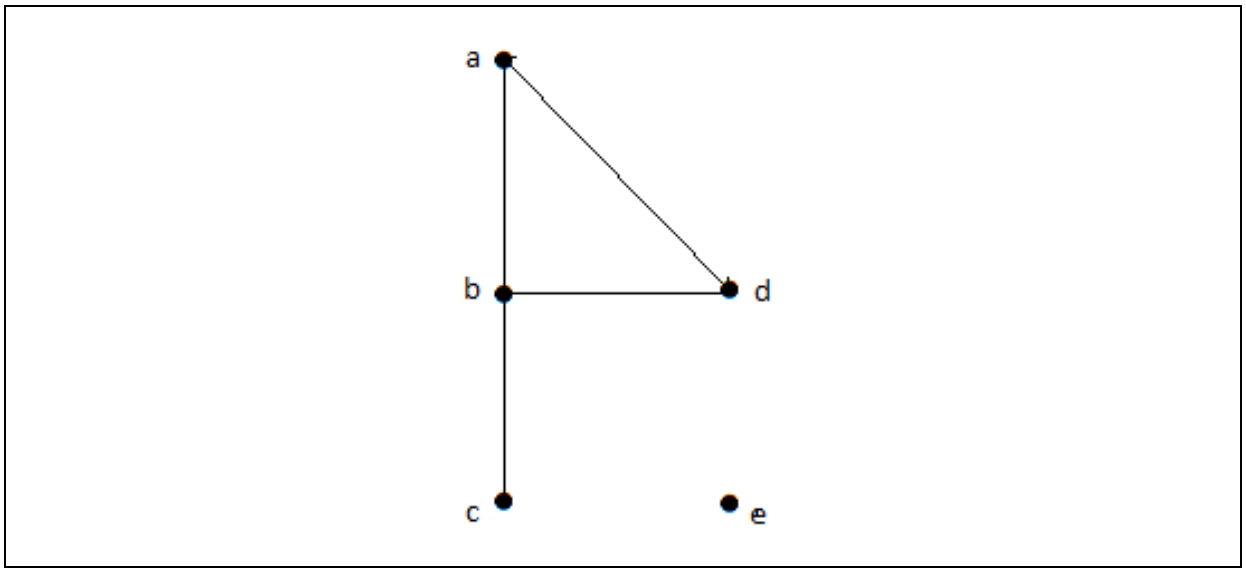
- Undirected Graph
- Directed Graph

Degree of Vertex in an Undirected Graph

An undirected graph has no directed edges. Consider the following examples.

Example 1

Take a look at the following graph:



In the above Undirected Graph,

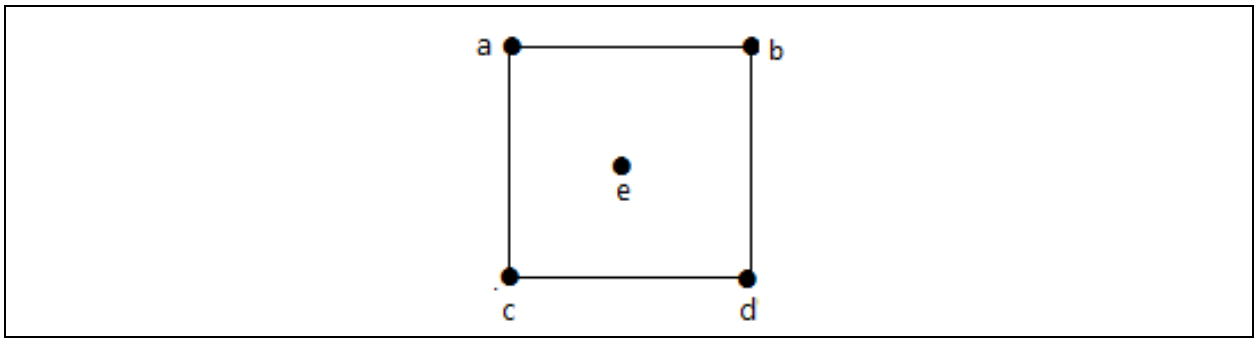
- deg(a) = 2, as there are 2 edges meeting at vertex ‘a’.
- deg(b) = 3, as there are 3 edges meeting at vertex ‘b’.
- deg(c) = 1, as there is 1 edge formed at vertex ‘c’

  So ‘c’ is a **pendent vertex**.
- deg(d) = 2, as there are 2 edges meeting at vertex ‘d’.
- deg(e) = 0, as there are 0 edges formed at vertex ‘e’.

  So ‘e’ is an **isolated vertex**.

Example 2

Take a look at the following graph:



In the above graph,

deg(a) = 2, deg(b) = 2, deg(c) = 2, deg(d) = 2, and deg(e) = 0.

The vertex ‘e’ is an isolated vertex. The graph does not have any pendent vertex.

Degree of Vertex in a Directed Graph

In a directed graph, each vertex has an **indegree** and an **outdegree**.

Indegree of a Graph

-
- Indegree of vertex V is the number of edges which are coming into the vertex V.
-
- **Notation:** $deg^+(V)$.

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